Calculus 2 - Test 2 Review

Dr. Graham-Squire, Spring 2013

1. Evaluate the integrals. Be sure to use correct notation where necessary and to show all of your work.

   (a) $\displaystyle\int_1^{\sqrt{2}} \frac{x^5}{\sqrt{4-x^2}}\,dx$ (Round to nearest .001)

   Ans: Use trig substitution (easier) or integration by parts (harder). Final answer is 0.758.

   (b) $\displaystyle\int_0^2 \frac{3}{\sqrt{2-x}}\,dx$

   Ans: This is an improper fraction because the function is not continuous at 2. Need to use limit notation to get full credit, final answer is $6\sqrt{2}$

2. Use the Midpoint Rule with six subintervals (M_6) to approximate $\displaystyle\int_0^3 \frac{dt}{1+t^2+t^4}$. Then use Sage, Maple or a graphing calculator to calculate the actual integral.

   Ans: 0.89548 for M_6, actual numerical integral is 0.89537.

3. Sketch the region enclosed by $y = x^3 - 9x$ and $y = -5x$ and then find its area. (Note: the answer is <u>not</u> zero).

   Ans: Need to calculate $\displaystyle\int_{-2}^0 [(x^3-9x)-(-5x)]dx + \int_0^2 [(-5x)-(x^3-9x)]dx$. Final answer is 8.

4. Consider the region W bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$ and $x = 3$. Find the volume of the solid obtained by rotating W about (a) the line $y = -3$ and (b) the y-axis.

   Ans: (a) Using washers, get $\displaystyle\int_1^3 \pi[(3+\frac{1}{x})^2 - 3^2]\,dx = \pi(6\ln 3 + (2/3))$.

   (b) Using shells, get $\displaystyle\int_1^3 2\pi x(\frac{1}{x})\,dx = 4\pi$.

5. Calculate the arc length of the curve $y = 4(x-3)^{3/2}$ for $3 \le x \le \frac{37}{12}$.

   Ans: Can take the integral by hand. final answer is 7/54.

6. A tank has the shape of an inverted circular cone with height 10 meters and base radius 4 meters. It is filled with water to a height of 8 meters. Note: The density of water is 1000 kg/cubic meter, and gravity is 9.8 m/sec^2.

   (a) Find the work required to empty the tank by pumping all of the water to the top of the tank.

   Ans: $1568\pi \displaystyle\int_0^8 x^2(10-x)dx = 107041.3\pi$

   (b) Find the work required to pump the water to a point that is 6 meters above the top of the tank.

Ans: $1568\pi \int_0^8 x^2(16-x)dx = 2760533.3\pi$

7. We have a cable that weighs 3 lbs/ft attached to a bucket filled with coal that weighs 700 lbs. The bucket is initially at the bottom of a 600 ft mine shaft. Answer each of the following.

   (a) Determine the amount of work required to lift the bucket to the midpoint of the shaft.

   Ans: Work for top half of rope + Work for bottom half of rope + work for bucket of coal = $\int_0^{300} 3x\,dx + 300(3)(300) + 700(300) = 615000$

   (b) Determine the amount of work required to lift the bucket from the midpoint of the shaft to the top of the shaft

   Ans: $\int_0^{300} 3x\,dx + 700(300) = 345000$

   (c) Determine the amount of work required to lift the bucket all the way up the shaft.

   Ans: $\int_0^{600} 3x\,dx + 700(600) = 960000.$